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μ problem in an extended gauge mediation supersymmetry breaking

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Abstract. We study the μ problem and radiative electroweak symmetry breaking in an extended gauge mediation supersymmetry breaking model, in which the messenger fields are assumed to couple to the different singlet fields due to the discrete symmetry. Since the spectrum of superpartners is modified, the constraint from the μ problem can be relaxed in comparison with the ordinary GMSB at least from the viewpoint of radiative symmetry breaking. We study the consistency of the values of μ and B_μ with radiative electroweak symmetry breaking and also the mass spectrum of the superpartners.

1 Introduction

Supersymmetry is now considered to be the most promising candidate for the solution of the gauge hierarchy problem. Although we have no direct evidence of the supersymmetry still now, the unification shown by the gauge couplings in the minimal supersymmetric standard model (MSSM) may indirectly reveal its signal. In the supersymmetric models the most important subject is to clarify the supersymmetry breaking mechanism in the observable world. Flavor changing neutral current processes severely constrain the scenario for the supersymmetry breaking. From this point of view, gauge mediation supersymmetry breaking (GMSB) [1–8] seems to be prominent, since the mediation is performed in a flavor blind way by the standard model gauge interactions.

In the ordinary minimal GMSB scenario [3–7], the messenger fields (q, ℓ) and $(\bar{q}, \bar{\ell})$ which come from the vectorlike chiral superfields $\mathbf{5} + \bar{\mathbf{5}}$ of SU(5) are considered to have couplings in the superpotential such as

$$W_{\rm GMSB} = \lambda_q S q \bar{q} + \lambda_\ell S \ell \bar{\ell}, \quad (1)$$

where S is a singlet chiral superfield.[1] Both the scalar component S and its F term F_S are assumed to get vacuum expectation values (VEVs) through the couplings to the fields in the hidden sector where supersymmetry is assumed to be broken. The masses of the gauginos and the scalar superpartners are respectively produced by the one-loop and two-loop effects through the couplings in (1). These masses are characterized by $\Lambda \equiv \langle F_S \rangle / \langle S \rangle$ and then Λ is considered to be in the range 20–100 TeV.

The chiral superfield S is usually considered not to have a direct coupling to the doublet Higgs chiral superfields H_1 and H_2 in the superpotential, although it can be the origin for both the μ term and the bilinear soft supersymmetry breaking parameter B_μ. The reason is that the relation $B_\mu = \Lambda\mu$ induced from such a coupling makes $|B_\mu|$ too large for the electroweak symmetry breaking under the assumption $\mu = O(100)\,$GeV. On the other hand, if we assume that $|B_\mu|$ has a suitable value for the electroweak symmetry breaking, the resulting small μ cannot satisfy the potential minimum condition.[2] Since Λ takes a large value as mentioned above, it makes both μ and B_μ difficult to take suitable values for the radiative symmetry breaking [3–5, 8]. Even if there is no such coupling in the superpotential, μ and B_μ can be produced radiatively picking up the supersymmetry breaking effect, and $B_\mu = \Lambda\mu$ is generally satisfied [9]. This suggests that the electroweak symmetry breaking cannot be induced radiatively also in this case. Thus, it is usually considered that the μ term should have another independent origin. This requires us to introduce new additional fields for this purpose. A lot of models of this kind have been proposed by now [3–5, 8, 9].

In this paper we show that the extended GMSB model proposed here may ease the difficulty of the μ problem in comparison with the ordinary GMSB at least from the viewpoint of radiative symmetry breaking. In this model we can relax the constraint on μ and B_μ. The consistency of such a scenario with radiative electroweak symmetry breaking is studied in some detail by solving numerically the renormalization group equations (RGEs). Its phenomenological results are also discussed. We present a concrete example in the appendix. In this model the superpotential is suitably arranged by the discrete symmetry which is introduced to resolve the doublet–triplet Higgs degeneracy [10] in the basis of the direct product gauge structure.

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[1] We will use the same notation for the scalar component as its chiral superfield.

[2] In the ordinary GMSB scenario the potential minimum condition requires $|B_\mu| < \mu^2$ as seen later.

2 Soft SUSY breaking and the μ problem

We extend the superpotential (1) for the messenger fields in such a way that the messenger fields $q, \bar{q}$ and $\ell, \bar{\ell}$ couple to the different singlet chiral superfields S_1 and S_2 which are assumed to have couplings to the hidden sector where the supersymmetry is supposed to be broken. This can happen incidentally as a result of a suitable discrete symmetry as we will see for an explicit example in the appendix. Thus the couplings of messenger fields are expressed by

$$W'_{\rm GMSB} = \lambda_q S_1 q\bar{q} + \lambda_\ell S_2 \ell\bar{\ell}\,. \quad (2)$$

If we assume that both S_α and F_{S_α} get the VEVs, the gaugino masses and the soft scalar masses are generated through one-loop and two-loop diagrams, respectively, as in the ordinary case. However, the mass formulas are modified from the ordinary ones since the messenger fields $q,\ \bar{q}$ and $\ell,\ \bar{\ell}$ couple to the different singlet fields S_α.

The mass formulas of the superpartners in this type of model have been discussed in [11]. Under the ordinary assumptions, such as $\langle F_{S_\alpha}\rangle \ll \lambda_{q,\ell}\langle S_\alpha\rangle^2$ [3], the mass formulas take a very simple form. The masses M_r of the gauginos λ_r of the MSSM gauge group can be written in the form of[3]

$$M_3 = \frac{\alpha_3}{4\pi}\Lambda_1\,, \quad M_2 = \frac{\alpha_2}{4\pi}\Lambda_2\,, \quad M_1 = \frac{\alpha_1}{4\pi}\left(\frac{2}{3}\Lambda_1 + \Lambda_2\right)\,, \quad (3)$$

where $\alpha_r = g_r^2/4\pi$ and $\Lambda_\alpha = \langle F_{S_\alpha}\rangle/\langle S_\alpha\rangle$. The soft scalar masses $\tilde{m}_f^2$ can be written as

$$\tilde{m}_f^2 = 2\left[C_3\left(\frac{\alpha_3}{4\pi}\right)^2 + \frac{2}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right]|\Lambda_1|^2 + 2\left[C_2\left(\frac{\alpha_2}{4\pi}\right)^2 + \left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right]|\Lambda_2|^2\,, \quad (4)$$

where $C_3 = 4/3$ and 0 for the SU(3) triplet and singlet fields, and $C_2 = 3/4$ and 0 for the SU(2) doublet and singlet fields, respectively. The hypercharge Y is expressed as $Y = 2(Q - T_3)$.

These formulas can give a rather different mass spectrum for the gauginos and the scalar superpartners in comparison with the ordinary GMSB scenario. The spectrum depends on the value of Λ_2/Λ_1. In fact, if we assume $\Lambda_1 < \Lambda_2$, the mass difference between the color singlet fields and the colored fields tends to be smaller than the one in the ordinary scenario at least at the supersymmetry breaking scale. As an example, we take $\Lambda_1 = 60\,\text{TeV}$ and $\Lambda_2 = 150\,\text{TeV}$ to show a typical spectrum of the superpartners at the supersymmetry breaking scale. The resulting spectrum is

$$\begin{aligned} &M_3 \simeq 415\,\text{GeV}\,, \quad M_2 \simeq 418\,\text{GeV}\,, \quad M_1 \simeq 166\,\text{GeV}\,, \\ &\tilde{m}_Q \simeq 851\,\text{GeV}\,, \quad \tilde{m}_U \simeq 690\,\text{GeV}\,, \quad \tilde{m}_D \simeq 682\,\text{GeV}\,, \\ &\tilde{m}_L \simeq 520\,\text{GeV}\,, \quad \tilde{m}_E \simeq 195\,\text{GeV}\,, \\ &m_1 = m_2 \simeq 520\,\text{GeV}\,, \end{aligned} \quad (5)$$

where m_1 and m_2 are the masses of the Higgs scalars that couple with the fields in the down and up sectors of quarks and leptons, respectively. These masses are somewhat affected by the running effect based on the renormalization group equations (RGEs). As discussed in [7], in the minimal GMSB model the soft supersymmetry breaking A_f parameters can also be expected to be induced through the radiative correction in such a way that

$$A_f \simeq A_f(\Lambda) + M_2(\Lambda)\left(-1.85 + 0.34|h_t|^2\right) + \dots\,, \quad (6)$$

where we should omit a term with h_t except for the top sector ($f = t$). Thus, even if $A_f(\Lambda) = 0$ is assumed, we can expect A_f to be generated through this effect.[4]

As mentioned in the introduction, the values of μ and B_μ are crucial for the electroweak symmetry breaking. Here we examine the effect of the introduction of a coupling $\lambda_\mu S_1 H_1 H_2$ in the superpotential. It can give a contribution to both μ and B_μ terms in the form as follows:

$$\mu = \lambda_\mu\langle S_1\rangle\,, \quad B_\mu = \lambda_\mu\langle F_{S_1}\rangle\,. \quad (7)$$

Unfortunately, as in the ordinary case the problematic relation $B_\mu = \mu\Lambda_1$ is satisfied also in this case. However, this relation does not eventually rule out the possibility for radiative symmetry breaking in the present case. This is very different from the ordinary GMSB.

An important aspect of the μ problem in the GMSB is crucially related to radiative electroweak symmetry breaking. In order to see this, we study the well-known conditions for the radiative electroweak symmetry breaking. In the MSSM the minimization conditions of the tree-level scalar potential are written as

$$\sin 2\beta = \frac{2B_\mu}{m_1^2 + m_2^2 + 2\mu^2}\,, \quad (8)$$

$$m_Z^2 = \frac{2m_1^2 - 2m_2^2\tan^2\beta}{\tan^2\beta - 1} - 2\mu^2\,, \quad (9)$$

where we assume that μ and B_μ are real, for simplicity. In these equations the Higgs scalar masses m_1^2 and m_2^2 should be improved into the values at the weak scale by using the RGEs. If we take account of the dominant one-loop contributions, they can be written as [7]

$$m_1^2(M_W) \simeq m_1^2(\Lambda) - \frac{3}{2}M_2^2(\Lambda)\left(\frac{\alpha_2(M_W)^2}{\alpha_2(\Lambda)^2} - 1\right) - \frac{1}{22}M_1^2(\Lambda)\left(\frac{\alpha_1(M_W)^2}{\alpha_1(\Lambda)^2} - 1\right)\,,$$

[3] If $\Lambda_{1,2}$ are complex, gaugino masses can have physical phases and they can affect various phenomena [12].

[4] In the present study we assume $A_f(\Lambda) = 0$. The soft supersymmetry breaking parameter B_μ/μ is also known to follow a radiative correction similar to (6) and phenomenological studies have been done [7,13,14]. However, we will discuss the other origin of $B_\mu(\Lambda)$ in the following.

$$m_1^2(M_W) - m_2^2(M_W) \simeq \frac{6h_t^2}{8\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\Lambda}{m_{\tilde{t}}}\right), \qquad (10)$$

where h_t and $m_{\tilde{t}}$ represent the top Yukawa coupling constant and the stop mass, respectively. They are approximated by the values at Λ. The masses of the gauginos and scalar superpartners at the supersymmetry breaking scale Λ are determined by (3) and (4).

We first recall the situation for radiative symmetry breaking in the ordinary GMSB case ($\Lambda_1 = \Lambda_2$) by checking the condition $m_1^2 + m_2^2 + 2\mu^2 > 2|B_\mu|$. It is obtained from the condition (8) and is also required by the vacuum stability. Inserting (10) into this inequality, we find that this necessary condition can be approximately written as

$$\left[\frac{3}{2}\left(\frac{\alpha_2}{4}\pi\right)^2 + \frac{5}{6}\left(\frac{\alpha_1}{4}\pi\right)^2\right]\left(2 - \frac{4h_t^2}{3\pi^2}\left(\frac{\alpha_3}{\alpha_2}\right)^2 \ln\frac{\sqrt{6}\pi}{\alpha_3}\right) > \frac{2}{\Lambda^2}(|B_\mu| - \mu^2). \qquad (11)$$

It is easy to find that the condition (11) is never satisfied unless h_t takes an unacceptably small value in the case of $|B_\mu| > \mu^2$, which is caused by the relation $B_\mu = \mu\Lambda$ because of $\Lambda \gg \mu$. Thus, we need to consider an additional origin of μ to make the condition $B_\mu < \mu^2$ be satisfied. This is the well-known result in the ordinary GMSB scenario [5,8]. This fact might make us consider that the condition $m_1^2+m_2^2+2\mu^2 > 2|B_\mu|$ cannot be satisfied in the GMSB model without the new origin for μ as far as the undesirable relation $B_\mu = \mu\Lambda$ exists. In the present model, however, Λ_1 is not generally supposed to be equal to Λ_2. This feature can give us a new possibility with regard to electroweak symmetry breaking even if the relation $B_\mu = \mu\Lambda_1$, if $\Lambda_1 < \Lambda_2$ is satisfied, and then the spectrum of the superpartners is modified.

In order to see this, it is useful to note that the factor $(\alpha_3/\alpha_2)^2 \ln(\sqrt{6}\pi/\alpha_3)$ in (11) should be modified into an approximated factor

$$\left[1 + \frac{16}{9}\left(\frac{\alpha_3}{\alpha_2}\right)^2\left(\frac{\Lambda_1}{\Lambda_2}\right)^2\right] \ln\frac{\sqrt{6}\pi}{(\alpha_3^2(\Lambda_1/\Lambda_2)^2 + 9\alpha_2^2/16)^{1/2}} \qquad (12)$$

in the present extended GMSB. This is caused by the change in the formulas of the soft scalar masses. We find that $m_1^2 + m_2^2 + 2\mu^2 > 2|B_\mu|$ is satisfied, as far as the condition $\Lambda_1 < \Lambda_2$ is fulfilled even in the case of $h_t \simeq 1$ and $|B_\mu| > \mu^2$.[5] The same change related to the radiative correction due to the top Yukawa coupling tends to make the allowed value of $\tan\beta$ smaller than the one in the ordinary GMSB scenario with the additional contribution to the μ term [5, 7, 14]. This can be found from (9). Moreover, the same equation suggests that there appears an upper bound of Λ_2/Λ_1 if we impose the lower bound for $\tan\beta$. For example, if we require $\tan\beta > 2$, we find that $\Lambda_2/\Lambda_1 \lesssim 3.5$ should be satisfied.

A more accurate analysis of this aspect can be done numerically by using the one-loop RGEs in the MSSM. For this purpose we can transform the conditions (8) and (9)

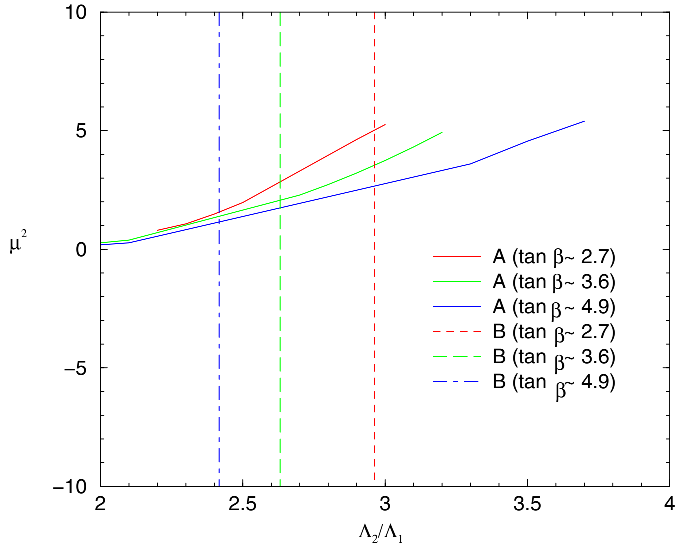

Fig. 1. The values of μ_A^2 and μ_B^2 predicted by the conditions for radiative symmetry breaking in the case of $B_\mu = \mu\Lambda_1$. We can find the solutions as the crossing points of μ_A^2 and μ_B^2

into the formulas for μ^2 such as

$$\mu_A^2 = \frac{1}{4\Lambda_1^2}\left[(m_1^2 - m_2^2)\tan 2\beta + m_Z^2 \sin 2\beta\right]^2,$$

$$\mu_B^2 = (m_1^2 - m_2^2)\frac{\tan^2\beta}{\tan^2\beta - 1} - m_1^2 - \frac{1}{2}m_Z^2, \qquad (13)$$

where we use $B_\mu = \mu\Lambda_1$. To estimate these formulas we take the following procedure. The gauge and Yukawa coupling constants are evolved from the gauge coupling unification scale to the weak scale. The soft supersymmetry breaking parameters are introduced at Λ_2 and evolved to the weak scale. We use the weak scale values of m_1^2 and m_2^2 obtained in this way and also the value of $\tan\beta$ which is determined by the top quark mass and the value of top Yukawa coupling obtained from the RGEs.

In Fig. 1 we plot each value of μ_A^2 and μ_B^2 in the (Λ_2/Λ_1, μ^2) plane for several values of $\tan\beta$ at $\Lambda_1 = 60$ TeV. μ_A^2 takes very small values of $O(1)$ GeV and the smaller $\tan\beta$ realizes the larger value of μ_A^2. μ_B^2 is very sensitive to the value of Λ_2/Λ_1 in comparison with μ_A^2. We can find that there are solutions in the region such as $\Lambda_2/\Lambda_1 \lesssim 3$ if we impose $\tan\beta \gtrsim 2.4$ which corresponds to the constraint from the neutral Higgs boson search. However, if we assume the larger value for Λ_1, we can obtain the solutions for the larger values of Λ_2/Λ_1.

Although radiative symmetry breaking can be found to occur just within the framework without adding any fields, we need to impose other phenomenological constraints for the scenario to be realistic. Since the absolute values of Λ_1 and μ are directly constrained by the experimental bounds for the masses of the gluino and the neutralino, we find that these values should be in the range

$$\Lambda_1 \gtrsim 20\,\text{TeV}, \qquad \mu \gtrsim 10^2\,\text{GeV}. \qquad (14)$$

This means that the present solution to the μ problem requires another contribution to the μ term to overcome

[5] In this paper we assume that $\Lambda_{1,2}$ and μ are positive.

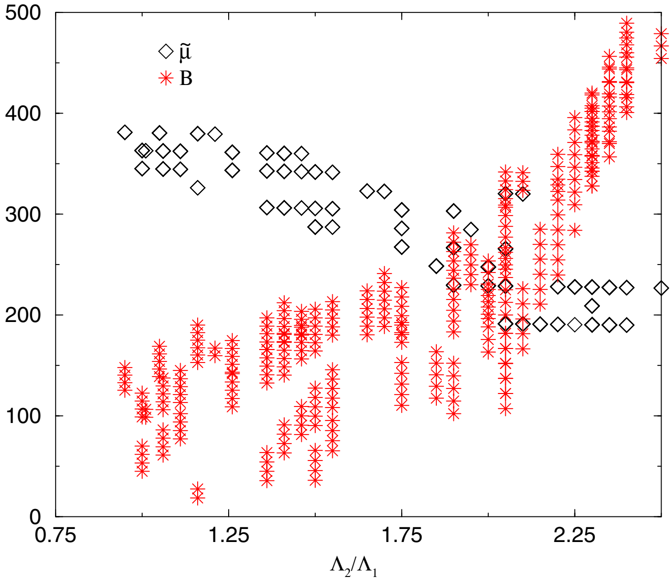

Fig. 2. The relation between $\tilde{\mu}$ and $B \equiv B_\mu/\tilde{\mu}$ in the solutions for the radiative symmetry breaking conditions. The $\tilde{\mu}$ and B are represented in GeV units

the constraint from the neutralino mass bound. However, it is useful to note that the situation on the origin of μ term is not the same as the ordinary case. Since $|B_\mu| < \mu^2$ is not required in this case unlike the ordinary GMSB, the constraint imposed from radiative symmetry breaking on the additional contribution μ' to the μ term can be expected to be sufficiently relaxed.

In order to study this aspect, we study radiative symmetry breaking and the spectrum of the superpartners by using the one-loop RGEs. In this study we need only modify the μ parameter into $\tilde{\mu} = \mu + \mu'$, where μ and B_μ are defined by (7). There are four free parameters and we take them as μ and μ' in addition to $\Lambda_{1,2}$. We can predict the spectrum of the superpartners in a rather restrictive way through this study. As the phenomenological constraints, we impose the experimental mass bounds for the superpartners and also require both the color and electromagnetic charge not to be broken. Under these conditions we search the allowed parameter region in the case of $\Lambda_1 = 60\,\text{TeV}$.

In Fig. 2 we give scatter plots of the solutions for radiative symmetry breaking. In this figure we can see that there are the solutions with $|B_\mu| > \tilde{\mu}^2$ for the $\Lambda_2/\Lambda_1 \gtrsim 2$ region, although the solutions are restricted into the ones with $|B_\mu| < \tilde{\mu}^2$ for the $\Lambda_2/\Lambda_1 \lesssim 2$ case. The ordinary GMSB should be noted to correspond to $\Lambda_2/\Lambda_1 = 1$. It should be also noted that the μ parameter can be smaller than the one in the ordinary GMSB.

We give the spectrum of the superpartners obtained in the same analysis for the case of $\Lambda_1 = 60\,\text{TeV}$ in Fig. 3. On the lightest chargino and neutralino by combining Figs. 2 and 3 we can find that they are dominated by the gaugino in the region $\Lambda_2/\Lambda_1 \lesssim 2$ and they change into the Higgsino dominated one in the region $\Lambda_2/\Lambda_1 \gtrsim 2$. The next lightest superpartner is always the neutralino as far as $\Lambda_2/\Lambda_1 > 1$ is assumed. The CP-even neutral Higgs boson mass slightly

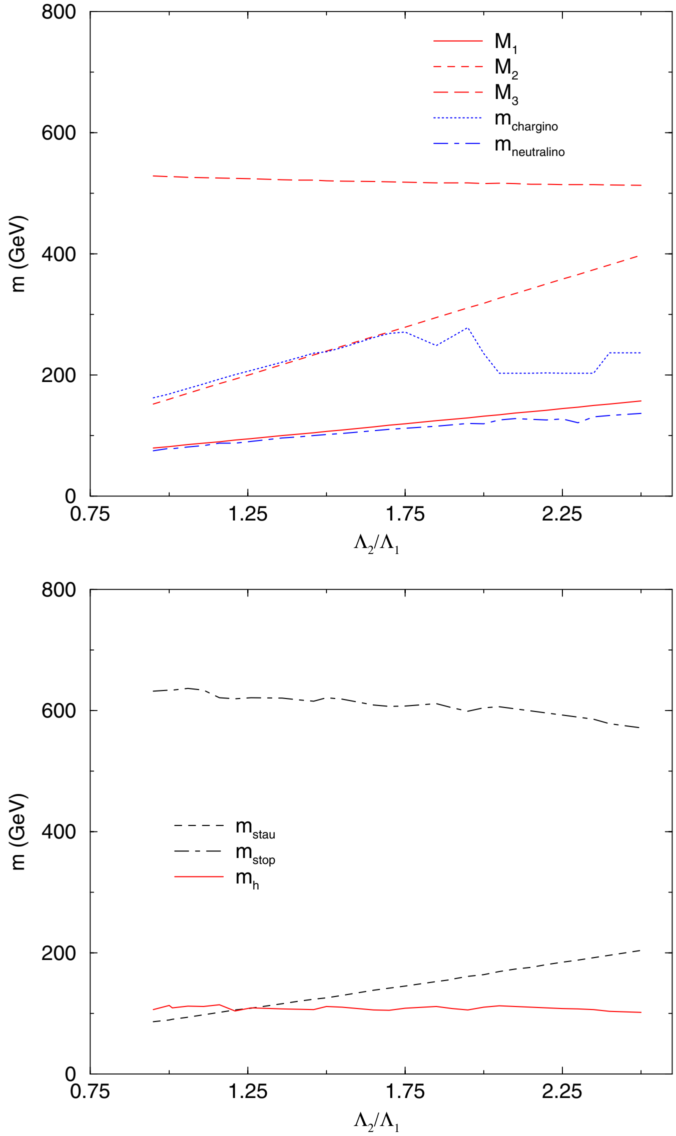

Fig. 3. Mass spectrum of the superpartners for the parameter sets which satisfy the radiative symmetry breaking conditions and various phenomenological constraints. The lightest one for each superpartner is shown except for the gauginos

decreases when Λ_2/Λ_1 increases. This is caused by the behavior of the stop mass. Although the neutral Higgs boson mass is almost equal to the experimental bound for this Λ_1 value, it can be larger by taking Λ_1 larger. The difference of the mass spectrum of superpartners from the one in the ordinary GMSB becomes clear in the larger Λ_2/Λ_1 region. In that region the mass difference between the colored fields and the color singlet fields becomes smaller and by using this feature we might distinguish the present model from the ordinary one.

Finally we briefly comment on other features. Since the soft masses of two Higgs fields H_1 and H_2 are the same at the supersymmetry breaking scale, radiative symmetry breaking predicts a relatively small value of $\tan\beta$ such as 2.5–7.5. Although $\tan\beta > 10$ is possible, it needs the fine tuning of parameters. There can appear an interesting feature for the coupling unification scale in the case of $\Lambda_2/\Lambda_1 > 1$. The unification scale of the coupling constants of SU(3) and SU(2) can be pushed up to the higher scale depending on the values of $\Lambda_{1,2}$.[6] This aspect comes from the fact that the SU(2) non-singlet superpartners decouple earlier than the others. However, we need further study whether the large shift of unification scale can be consistent with radiative symmetry breaking.

3 An additional contribution to the μ term

In this section we consider the embedding of the scenario studied above into the realistic model, which is defined by the following effective superpotential which can be induced from the construction given in the appendix:

$$W_2 = h_1\Psi_{\mathbf{10}}\Psi_{\mathbf{10}}H_2 + \frac{h_2\Phi_1}{M}\Psi_{\mathbf{10}}\Psi_{\bar{\mathbf{5}}}H_1 + \frac{h_3\Phi_1}{M_{\rm pl}}S_1H_1H_2$$
$$+h_4\Psi_{\mathbf{1}}\Psi_{\bar{\mathbf{5}}}H_2 + \frac{h_5\Phi_1\Phi_2}{M_{\rm pl}}\Psi_{\mathbf{1}}\Psi_{\mathbf{1}}$$
$$+\frac{\lambda_1\Phi_2}{M}S_1q\bar{q} + \frac{\lambda_2\Phi_2}{M}S_2\ell\bar{\ell}\,, \qquad (15)$$

where we use the notation of the fields given in Table 1 of the appendix. M and $M_{\rm pl}$ are the effective unification scale of $O(10^{16})$ GeV and the reduced Planck scale, respectively.[7] We use the usual notation in the MSSM for the Higgs fields such as $H_1 \equiv \tilde{H}_{\bar{\mathbf{2}}}$ and $H_2 \equiv H_{\mathbf{2}}$. The several terms can be suppressed by the additional factors $\epsilon_{1,2} \equiv \langle\Phi_{1,2}\rangle/M$ coming from the VEVs $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle$ given in (32), since each term is controlled by the discrete symmetry. This feature makes several terms phenomenologically favorable. For example, the second term in the first line which includes the MSSM relevant terms seems to be favorable to explain the hierarchy between the masses of top and bottom quarks for the various values of $\tan\beta$. The mass hierarchy between the top quark and the bottom quark requires that ϵ_1 should be $O(10^{-2})$ or larger. This feature also causes the favorable effects on the second line which is relevant to the neutrino masses. If the VEVs $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle$ take the suitable values so as to be $\epsilon_1 = O(10^{-2})$ and $\epsilon_2 = O(1)$, the right-handed neutrinos $\Psi_{\mathbf{1}}$ can have the mass of $O(10^{13})$ GeV which is suitable to explain the experimental data for the solar and atmospheric neutrinos.

In the last line of (15), as we expected, the messenger fields q, $\bar{q}$ and ℓ, $\bar{\ell}$ couple with the different singlet fields $S_{1,2}$. Thus the messenger sector assumed in the previous discussion is realized. The last term in the first line can be the origin of the μ and B_μ terms, since both the scalar component and the F-component of S_1 are assumed to get the VEVs. Both μ and B_μ in (7) can be induced by taking $\lambda_\mu \simeq h_3\epsilon_1 M/M_{\rm pl}$. In fact, if we assume

$$\epsilon_1 = O\left(10^{-2}\right)\,, \quad \epsilon_2 = O\left(1\right)\,, \quad \langle S_1\rangle = O\left(10^5\right)\ {\rm GeV}\,,$$
$$\langle F_{S_1}\rangle = O\left(10^9\right)\ {\rm GeV}^2\,, \qquad (16)$$

we can consistently obtain a suitable value of μ such as $O(1)$ GeV for radiative electroweak symmetry breaking as has been discussed in the previous section. To satisfy the mass bounds for the chargino and the neutralino, however, we need to introduce an additional origin for μ'. If we can introduce such an origin as $\mu' = O(100)$ GeV, the radiative symmetry breaking condition is expected to be easily satisfied based on the present analysis.

The new origin may be given by the non-renormalizable couplings among the Higgs chiral superfields and the singlet chiral superfield N whose scalar potential has a negative curvature due to the Kähler potential interaction [4,8]. We consider the following terms in the effective Lagrangian:

$$\int {\rm d}^4\theta S_1^\dagger S_1 N^\dagger N \qquad (17)$$
$$+\left\{\int {\rm d}^2\theta\left(\frac{1}{M_{\rm pl}}N^4 + \frac{\Phi_1}{M_{\rm pl}}NH_1H_2\right) + {\rm h.c.}\right\}\,,$$

where each term should be determined by the discrete symmetry presented in Table 1. In fact, the last two terms can exist as the $\mathcal{G}\times F$ and F' invariant ones if the F charge of N is assigned as $\omega = 5$. From these terms the additional contribution μ' to the μ term is yielded at the tree-level as

$$\mu' \simeq \epsilon_1\sqrt{F_{S_1}}\,. \qquad (18)$$

If we use (16), we can obtain μ' of $O(100)$ GeV. Since B_μ is not produced at the tree-level following this μ' generation, the dominant B_μ comes from $\Phi_1S_1H_1H_2$ in (15), and then our result for radiative symmetry breaking, obtained in the previous section, can be directly applicable to this model.

Finally we should comment on the mass eigenvalues and the mixings of quarks and leptons in this model. We would like to stress that the existence of the suppression factor ϵ_1 is favorable for the explanation of the masses of quarks and leptons as mentioned below (15). The value of ϵ_1 is constrained by the masses of the bottom quark and the τ lepton. If we impose $\tan\beta \gtrsim 2$, which is required by the constraint from the neutral Higgs boson mass, $\epsilon_1 \gtrsim 10^{-2}$ should be satisfied. This is consistent with the condition given in (16) and also with the neutrino oscillation data. Moreover, if we introduce the flavor dependence of the F charge and also the Frogatt–Nielsen flavor U(1) symmetry into this model along the line of [18], the qualitatively satisfactory mass eigenvalues and mixing angles for the quarks and the leptons are expected to be derived. We will discuss this subject in another place.

4 Summary

We have investigated the μ problem and radiative symmetry breaking in the extended GMSB scenario, in which the SU(3) triplet messenger and the SU(2) doublet one couple to the different singlet chiral superfields. Some kind of discrete symmetry is assumed to force the SU(3) triplet messenger and the SU(2) doublet messenger respectively

[6] The similar possibility has been discussed in other context in [15].

[7] Although the effectively induced F' invariant non-renormalizable terms are expected to be suppressed by M, the F invariant ones are suppressed by $M_{\rm pl}$.

to couple to the different singlet fields. The direct coupling between the doublet Higgs fields and the one of these singlet fields is assumed to be allowed but suppressed largely due to this discrete symmetry. This coupling can give the origin of both μ and B_μ terms. Since the model has two scales which are relevant to the supersymmetry breaking and the superpartner masses depend on both of them, the induced μ and B_μ can be consistent with radiative electroweak symmetry breaking. This aspect is largely different from the ordinary minimal GMSB and it may present a new solution for the μ problem in the GMSB scenario at least from the viewpoint of the radiative symmetry breaking. However, to make the model consistent with the experimental bounds for the masses of superpartners, it seems to be required to introduce the additional contribution to the μ term. Some interesting features different from the ordinary GMSB appear in the spectrum of the superpartners and induce new phenomenological features. The mass difference between the colored and color singlet superpartners tends to be smaller in comparison with the ordinary GMSB scenario. The gaugino masses become non-universal generally. The next lightest superparticle can be always the neutralino. As a result of these features, the gauge coupling unification scale may be pushed upwards somewhat.

Further phenomenological study of this kind of model seems to be worthwhile since it can be related to the reasonable motivation such as the doublet–triplet splitting problem in the grand unified model.

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Appendix

In this appendix we present the construction of the model which can realize the extended GMSB discussed in the text. As its starting point we consider a model with a direct product gauge structure such as $\mathcal{G} = \mathrm{SU}(5)' \times \mathrm{SU}(5)''$ and a global discrete symmetry F which commutes with this gauge symmetry [11]. The field content of the model is listed in Table 1.

The model is constructed based on the following deconstruction [11]. The theory space before the deconstruction is represented by the moose diagram which is composed of the n sites Q_i placed on the vertices of an n-polygon and one site on its center P of this polygon [16]. We assign $\mathrm{SU}(5)'$ on the site P and $\mathrm{SU}(5)''$ on each site Q_i and also put a bifundamental chiral superfield $\varPhi_i$ on each link from P to Q_i. On each link from Q_i to Q_{i+1} we put the adjoint Higgs chiral superfield $\varSigma$ of $\mathrm{SU}(5)''$. We introduce an equivalence relation only for the boundary points of the polygon by a $2\pi/n$ rotation and we identify this Z_n symmetry with the above mentioned discrete symmetry F. This makes us consider the reduced theory space composed of only three sites P, Q_1 and Q_2, in which the field contents become equivalent to the one given in Table 1. Under these settings the remaining symmetries can be determined through the following discussion [11,16].

We consider the unitary link variables $U_i \equiv \exp(\mathrm{i}\phi_i/\tilde{M})$ and $\mathcal{W} \equiv \exp(-\mathrm{i}\sigma/\tilde{M})$ where $\phi_{1,2}$ and σ are the scalar components of $\varPhi_{1,2}$ and $\varSigma$. If we use $\mathcal{W}$ as introduced above, this equivalence relation requires that $\mathcal{W}^n = 1$ is satisfied. Thus we can write $\mathcal{W}$ as follows:

$$\mathcal{W} = \mathrm{diag}\left(\mathrm{e}^{2\mathrm{i}\rho},\ \mathrm{e}^{2\mathrm{i}\rho},\ \mathrm{e}^{2\mathrm{i}\rho},\ \mathrm{e}^{-3\mathrm{i}\rho},\ \mathrm{e}^{-3\mathrm{i}\rho}\right)\,, \tag{19}$$

where $\mathrm{e}^{\mathrm{i}\rho}$ is the nth root of unity. If we assume that our model is obtained as a result of this deconstruction, the condition

$$U_i \mathcal{W} U_{i+1}^{-1} = 1 \tag{20}$$

Table 1. Charge assignment of the discrete symmetry F' for the chiral superfields. For the adjoint Higgs field $\varSigma$ we show only the F' charge of the diagonal elements

	$\mathcal{F}(\mathcal{G}$ rep.)	F	F'	
			$\mathbf{3}\in\mathbf{5}$ or $\bar{\mathbf{3}}\in\bar{\mathbf{5}}$	$\mathbf{2}\in\mathbf{5}$ or $\bar{\mathbf{2}}\in\bar{\mathbf{5}}$
Quarks/Leptons	$\varPsi_{\mathbf{10}}^j(\mathbf{10},\mathbf{1})$	α	α	α
($j=1\sim 3$)	$\varPsi_{\bar{\mathbf{5}}}^j(\bar{\mathbf{5}},\mathbf{1})$	β	β	β
	$\varPsi_{\mathbf{1}}^j(\mathbf{1},\mathbf{1})$	γ	γ	γ
Higgs fields	$H(\mathbf{5},\mathbf{1})$	ρ	ρ	ρ
	$\tilde{H}(\mathbf{1},\bar{\mathbf{5}})$	ξ	$\xi+2a$	$\xi-3a$
Messenger fields	$\bar{\chi}(\bar{\mathbf{5}},\mathbf{1})$	δ	δ	δ
	$\chi(\mathbf{1},\mathbf{5})$	ϵ	$\epsilon-2a$	$\epsilon+3a$
Bifundamental field	$\varPhi_1(\bar{\mathbf{5}},\mathbf{5})$	η	$\eta+2b$	$\eta-3b$
	$\varPhi_2(\mathbf{5},\bar{\mathbf{5}})$	ζ	$\zeta-2b$	$\zeta+3b$
Adjoint Higgs field	$\varSigma(\mathbf{1},\mathbf{24})$	0	0 (for $\varSigma_i^{\bar{i}}$)	
Singlets	$S_1(\mathbf{1},\mathbf{1})$	θ	θ	
	$S_2(\mathbf{1},\mathbf{1})$	τ	τ	
	$N(\mathbf{1},\mathbf{1})$	ω	ω	

should be satisfied for $i = 1$, which means that the holonomy around each two-dimensional plaquette is equal to 1.[8]

Now we consider the transformation property of this vacuum under the gauge transformation such as

$$U_i' = \omega'\, U_i\, (\omega'')^{-1}\,, \qquad \mathcal{W}' = \omega''\, \mathcal{W}\, (\omega'')^{-1}\,, \tag{21}$$

where ω' and ω'' are the group elements of $\mathrm{SU}(5)'$ and $\mathrm{SU}(5)''$, respectively. The invariance of U_i and $\mathcal{W}$ shows that the group elements ω of the unbroken gauge group satisfy the condition: $\omega = \omega' = \omega''$ and $[\omega,\ \mathcal{W}] = 0$. Since we take the VEVs of Higgs scalar fields as (19) and (20), the unbroken gauge group is $\mathcal{H} = \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ which is a subgroup of the diagonal sum SU(5) of $\mathcal{G}$. Next we introduce a discrete symmetry F' as a diagonal subgroup of $F \times G_{\mathrm{U}(1)''}$ where $G_{\mathrm{U}(1)''}$ is a discrete subgroup of a hypercharge in $\mathrm{SU}(5)''$. If we write the group elements of F and $G_{\mathrm{U}(1)''}$ as f and ω_D, the transformation of U_i due to F' can be written as

$$U_i' = (fU_i)\,\omega_D^{-1} = U_{i+1}\omega_D^{-1}\,. \tag{22}$$

If we take ω_D as $\mathcal{W}$, we find that U_i is invariant under this transformation due to the relation $U_i\mathcal{W}U_{i+1}^{-1} = 1$ and F' remains unbroken. The invariance of $\mathcal{W}$ is also clear. Thus we can conclude that in this model the symmetry $\mathcal{G}\times F$ breaks down into $\mathcal{H}\times F'$ by considering the vacuum defined by (19) and (20). In the following we suppose that the model is defined by this $\mathcal{H}\times F'$, that is, the model is assumed not to be SU(5) symmetric. The situation is the same as in the case of the heterotic string with the Wilson line [17]. An important point is that every field which has a non-trivial transformation property with respect to $\mathrm{SU}(5)''$ can have different charges for the doublet and the triplet, since the definition of F' contains the discrete subgroup of $\mathrm{U}(1)''$ in $\mathrm{SU}(5)''$ as its component.[9] We give the charges assignment for the fields with respect to F' in Table 1.

In order to fix the discrete charges for F' we impose the following conditions on F' to satisfy various phenomenological constraints in a way similar to as we have followed in [11] .

(i) To realize the doublet–triplet splitting, only the color triplet Higgs chiral superfields $H_{\mathbf{3}}$ and $\tilde{H}_{\mathbf{3}}$ except for the ordinary doublet Higgs chiral superfields $H_{\mathbf{2}}$ and $\tilde{H}_{\mathbf{2}}$ should become massive after the symmetry breaking due to (32). All of the effective Yukawa couplings $\Phi_{1,2}H_{\mathbf{2}}\tilde{H}_{\mathbf{2}}$ and $\Sigma H_{\mathbf{2}}\tilde{H}_{\mathbf{2}}$ should be forbidden by F', although $\Sigma H_{\mathbf{3}}\tilde{H}_{\mathbf{3}}$ is allowed at least.[10] In terms of the discrete symmetry F' this gives the conditions on the effective superpotential such as

$$\begin{aligned}&\rho+\xi+2a=0\,,\quad \rho+\xi-3a\neq 0\,,\\ &\rho+\xi+\eta-3(a+b)\neq 0\,,\\ &\rho+\xi+\zeta-3(a-b)\neq 0\,.\end{aligned} \tag{23}$$

(ii) Yukawa couplings of quarks and leptons, that is, $\Psi_{\mathbf{10}}\Psi_{\mathbf{10}}H_{\mathbf{2}}$ and $\Psi_{\mathbf{10}}\Psi_{\bar{\mathbf{5}}}\tilde{H}_{\mathbf{2}}\Phi_1$ should exist under F'. This requires

$$2\alpha+\rho=0\,,\qquad \alpha+\beta+\xi+\eta-3(a+b)=0\,. \tag{24}$$

(iii) The fields χ and $\bar{\chi}$ should be massless at the $\mathcal{G}$ breaking scale and play the role of the messenger fields of the supersymmetry breaking which is assumed to occur in the S_α sector. These require the absence of $\Phi_{1,2}\chi\bar{\chi}$ and $\Sigma\chi\bar{\chi}$ under F'. We also impose the existence of the coupling $\Phi_2 S_\alpha\chi\bar{\chi}$ under F'. These conditions can be written as

$$\begin{aligned}\delta+\epsilon+\zeta+\theta-2(a+b)&=0\,,\\ \delta+\epsilon+\zeta+\tau+3(a+b)&=0\,,\\ \delta+\epsilon+\zeta-2(a+b)&\neq 0\,,\\ \delta+\epsilon+\eta-2(a-b)&\neq 0\,,\\ \delta+\epsilon-2a&\neq 0\,,\\ \delta+\epsilon+\zeta+3(a+b)&\neq 0\,,\\ \delta+\epsilon+\eta+3(a-b)&\neq 0\,,\\ \delta+\epsilon+3a&\neq 0\,.\end{aligned} \tag{25}$$

(iv) The neutrino should be massive and the proton should be stable. This means that $\Psi_{\bar{\mathbf{5}}}\Psi_{\mathbf{1}}H_{\mathbf{2}}$ and $\Phi_1\Phi_2\Psi_{\mathbf{1}}^2$ should exist and $\Psi_{\mathbf{10}}\Psi_{\bar{\mathbf{5}}}^2$ and $\Psi_{\mathbf{10}}^3\Psi_{\bar{\mathbf{5}}}$ should be forbidden [16]. These require

$$\beta+\gamma+\rho=0\,,\quad \eta+\zeta+2\gamma=0\,,\quad \alpha+2\beta\neq 0\,,\quad 3\alpha+\beta\neq 0\,. \tag{26}$$

(v) The gauge invariant bare mass terms of the fields such as $\Psi_{\bar{\mathbf{5}}}H$, $H\bar{\chi}$, $\tilde{H}\chi$ should be forbidden.[11] These conditions are summarized as

$$\beta+\rho\neq 0\,,\quad \rho+\delta\neq 0\,,\quad \xi+\epsilon\neq 0\,. \tag{27}$$

Here we additionally assume that both the origin of μ and B_μ is in the Higgs coupling with S_1 in order to embed our scenario discussed in the previous section into

[8] This corresponds to the energy minimum condition from the viewpoint of the lattice gauge [16].

[9] This is possible if we assume that $\mathrm{SU}(5)''$ is an induced symmetry as the diagonal sum of two SU(5) groups and also $\tilde{H}, \chi$ and Φ_1, Φ_2 are the fundamental representations of these different SU(5)s, respectively. In that case the charge normalization of the discrete group $G_{\mathrm{U}(1)''}$ can be independent from each other for $\tilde{H}$, χ and Φ_1, Φ_2. Thus the discrete parameters a and b in Table 1 can be determined independently up to the modulus n. Even if we do not refer to this kind of $\mathrm{SU}(5)''$, its discrete charges for $\tilde{H}$, χ and $\Phi_{1,2}$ can be different from each other even in the case of the same representation of $\mathrm{SU}(5)''$, since $G_{\mathrm{U}(1)''}$ is not just the hypercharge $\mathrm{U}(1)''$ but its discrete subgroup.

[10] This phenomenon suggesting the existence of F' may be understood by considering a partial cancellation between the direct coupling $\Phi_1 H\tilde{H}$ and a mass term $M\exp \mathrm{i}\{(-\phi_1+\sigma+\phi_2)/\tilde{M}\}H\tilde{H}$ induced by a holonomy operator associated with the symmetry breaking introduced to define the theory.

[11] We cannot forbid the bare mass terms of the singlet chiral superfields completely based on the discrete symmetry F' alone. Although we might need additional symmetry to prohibit it, we do not discuss this further here, and we only assume that they have no bare masses.

this unified model. For this realization we introduce an F invariant term $\Phi_1 S_1 H\tilde{H}$ in the superpotential before the $\mathcal{G}$ breaking.[12] The condition for the existence of such a term can be written as

$$\rho + \xi + \eta + \theta = 0 \,. \tag{28}$$

Every condition above should be understood up to the modulus n when we take $F' = Z_n$.

We can easily find an example of the consistent solution for these constraints, (23)– (28). In order to show its existence concretely, we give an example here. If we take $F' = Z_{20}$, these condition can be satisfied under the charge assignment,[13]

$$\begin{aligned}&\alpha = \epsilon = -1\,, \quad \rho = \theta = 2\,, \quad \tau = a = -3\,, \quad \xi = b = 4\,,\\ &\delta = -7\,, \quad \beta = \zeta = -\eta = 8\,, \quad \gamma = 10\,.\end{aligned} \tag{29}$$

After the symmetry breaking described by (32) the various F' invariant terms effectively appear as we have required. It should be noted that the different singlet fields $S_{1,2}$ are generally required for the couplings to χ and $\bar{\chi}$, which play the role of messengers of the supersymmetry breaking. This feature incidentally comes from the introduction of the direct product gauge structure motivated to realize the doublet–triplet splitting, which requires the F' charges of χ and $\bar{\chi}$ to satisfy

$$\theta - \tau = 5(a + b) \neq 0 \qquad (\mathrm{mod}\ n)\,. \tag{30}$$

In order to describe the effective low energy theory after the symmetry breaking due to the Wilson line, we may consider an F' invariant renormalizable superpotential such as

$$\begin{aligned}W_1 = {}& M_\phi \mathrm{Tr}\,(\Phi_1\Phi_2) + \frac{1}{2}M_\sigma \mathrm{Tr}\left(\Sigma^2\right)\\ &+\lambda\ \mathrm{Tr}\left(\Phi_1\Sigma\Phi_2 + \frac{1}{3}\Sigma^3\right)\,.\end{aligned} \tag{31}$$

As it is shown in [11], the scalar potential derived from this W_1 has a non-trivial minimum which is realized at

$$\begin{aligned}&\sigma = \tilde{M}\ \mathrm{diag}\ (2,\ 2,\ 2,\ -3,\ -3)\,, \quad \phi_1 = \kappa\sigma\,,\\ &\phi_2 = \frac{1}{\kappa}\left(\frac{M_\sigma}{M_\phi} - 1\right)\sigma\,,\end{aligned} \tag{32}$$

where $\tilde{M}$ is defined as $\tilde{M} = M_\phi/\lambda$. These VEVs satisfy (19) and (20).[14] The low energy theory should be considered on the vacuum defined by (32).

[12] If we make the Higgs doublets couple to S_2 instead of S_1, μ seems not to be large enough to satisfy the radiative symmetry breaking condition.

[13] We have not taken account of the anomaly of F' here. Although this anomaly cancellation might require the introduction of new fields and impose the additional constraints on the charges, it will not affect the result of the present study of the model.

[14] The parameter κ is fixed by the above mentioned holonomy condition which can be expressed as $\kappa^2 - \kappa + 1 - M_\sigma/M_\phi = 0$.

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